

20100 Calculus I, Spring 2003

Practice for Final Test, due Thursday, May 15 for extra credit

1. Find  $\lim_{x \rightarrow -1} \frac{(x+1)^{\frac{3}{2}}}{\sin(x^2-1)}$
2. Find  $f'(x)$  and  $f'(2)$

$$f(x) = 5\sqrt{6-x} + \frac{2}{\sqrt[3]{2x^2}}$$

3. Find  $y'$   
(a)

$$y = \frac{3x-1}{x^3+x}$$

(b)

$$y = (3x+1)\tan^2\left(\frac{x}{2}\right)$$

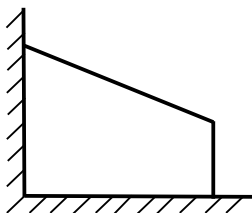
4. You are inflating a ball with the rate  $1 \text{ in}^3/\text{sec}$ . How fast is the volume of the ball increasing when the ball size (diameter) is 2ft?
5. Let

$$f(x) = \begin{cases} -ax^3, & x < 0 \\ x, & 0 \leq x \leq 4 \\ b + 4\sqrt{x}, & x > 4 \end{cases}$$

- (a) Find  $a$  and  $b$  so that  $f(x)$  is continuous for all  $x$
  - (b) Using your values of  $a$  and  $b$  find  $f'(0)$  or explain why it does not exist.
  - (c) Using your values of  $a$  and  $b$  find  $f'(4)$  or explain why it does not exist.
6. Let  $f(x) = x^3$ 
    - (a) Find the slope of the straight line connecting the points on the graph of  $f(x)$  where  $x = 2$  and  $x = 3$ .
    - (b) Use a definition of derivative (a limit process) to find  $f'(x)$ . (No credit for using the formul  $(x^r)' = rx^{r-1}$ )
    - (c) Use differential to approximate  $(2.2)^3$
  7. Let  $y$  be defined implicitly as a function of  $x$  by the equation  $\frac{\sin y}{x^2} + y \sin \pi x = 1$ .

- (a) Find  $\frac{dy}{dx}$  as a function of  $x$  and  $y$ .

- (b) Find  $\frac{dy}{dx}$  when  $x = 1$   
 (c) Use differentials to approximate  $y$  when  $x = 0.95$
8. You want to build a granary of volume  $10m^3$  and you are going to attach it to the existing wall. The height of the granary near that wall should be twice of the height of the opposite (front) side of the granary. Find the dimensions of the granary with minimal surface.



9. Sketch a graph of  $f(x) = \frac{x+1}{x^2+x+1}$ . Show all intercepts, local min/max, vertical and horizontal asymptotes, inflection points. Indicate concavity.
10. Find the integrals

(a)

$$\int_0^{\pi} \sin(2-x) dx$$

(b)

$$\int \frac{\sqrt[3]{\frac{1}{x^2} + 1}}{x^3} dx$$

(c)

$$\int \frac{(1 + \tan x)^3}{\cos^2 x} dx$$

11. Draw a sketch of the functions  $y = |x|$  and  $y = 2 - x^2$ . Find the area of the region bounded by the two curves.
12. The speed of the tired runner is  $v(t) = 1 + \frac{1}{\sqrt{t}}$  m/sec. Find the distance  $s(t)$  surmounted by the runner after  $t$  seconds.